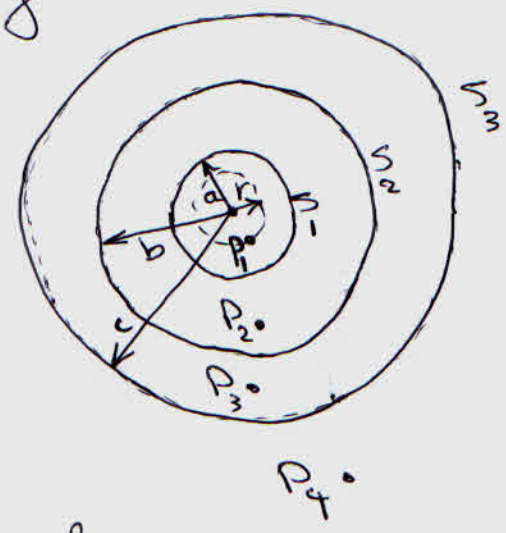
ex Find the electric field Density at P_1, P_2, P_3, P_4



$$\int_S D \cdot ds = Q|_{\text{inside } S}$$

$\int_{S_1} = 5\mu C$

$\int_{S_2} = 6\mu C$

$\int_{S_3} = 7\mu C$

at Point P_1 : $\boxed{r < a}$

$$\int_S D_1 \cdot ds = Q|_{\text{inside}}$$

$$D_1 (4\pi r^2) = 0$$

$$D_1 = 0$$

at Point P_2 : $\boxed{a < r < b}$

$$D_2 (4\pi r^2) = \int_{S_1} (4\pi a^2)$$

$$D_2 = \frac{\int_{S_1} a^2}{r^2}$$

at Point P_3 $\boxed{b < r < c}$

$$D_3 (4\pi r^2) = \int_{S_1} (4\pi a^2) + \int_{S_2} (4\pi b^2)$$

$$D_3 = \frac{\int_{S_1} a^2 + \int_{S_2} b^2}{r^2}$$

at Point P_4 $\boxed{r > c}$

$$D_4 (4\pi r^2) = \int_{S_1} (4\pi a^2) + \int_{S_2} (4\pi b^2) + \int_{S_3} (4\pi c^2)$$

$$D_4 = \frac{\int_{S_1} a^2 + \int_{S_2} b^2 + \int_{S_3} c^2}{r^2}$$

* Maxwell's 1st equation :- $\nabla \cdot \bar{D} = \rho_v$

ex In a certain material of $\epsilon_r = 4$ and

$D = 5xy\,\bar{a}_x + 6yz\,\bar{a}_y + 8z\,\bar{a}_z \ C/m^3$

find the volume charge density (ρ_v) at (5,0,3)

$$\nabla \cdot \bar{D} = \rho_v$$

$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= 5y + 6z + 8$$

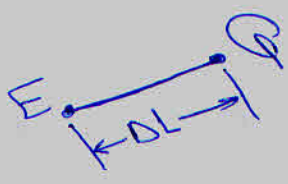
at $P = (5,0,3)$

$$\rho_v = 5(0) + 6(3) + 8 \quad C/m^3$$

$$= 26 \quad C/m^3$$

Chapter 4 :- Energy and Potential

$$\bar{E} = \frac{\bar{F}}{Q} \Rightarrow \bar{F} = \bar{E}\,Q$$



لو عندي مجال كهربي $\bar{E}$ وشحنة Q القوة المؤثرة عليها هي شحنة نتيجة المجال $\bar{E}$ هي $\bar{F}$

* لو عايز أحرك الـ Q مسافة معينة DL هشغل شغل (W)

work done (W)

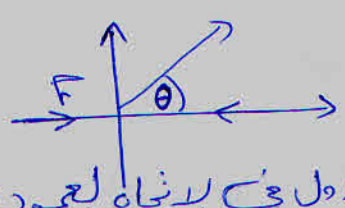
$$dW = \text{Force} * \text{distance}$$

$$= -Q\,\bar{E} * \Delta L$$

$$= -Q\,\bar{E} \cdot \Delta L$$

$$dW = -Q\,\bar{E} \cdot d\bar{L}$$

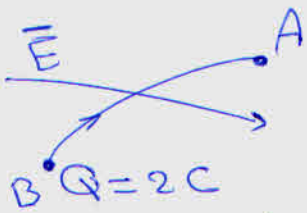
$$W = \int_{\text{intial}}^{\text{Final}} dW = -Q \int_{in}^{H} E \cdot dL$$



* الشغل المبذول في الاتجاه العمودي على اتجاه القوة يساوي صفر

* الشغل أكبر ما يمكن عندما تكون $\cos\theta = 0$

Ex Find the work done expanded in carrying 2C From B(1,0,1) to A(,8, ,6, 1) along the shorter arc of the circuit $x^2+y^2=1$, $z=1$ in an electric field $\bar{E}=y\,\bar{a_x}+x\,\bar{a_y}+2\,\bar{a_z}$



$$w=-Q\int_{in}^{F}\bar{E}\cdot d\bar{l}=$$

$$-2\int_{B}^{A}(y\,\bar{a_x}+x\,\bar{a_y}+2\,\bar{a_z})\cdot(dx\,\bar{a_x}+dy\,\bar{a_y}+dz\,\bar{a_z})$$

$$=-2\left[\int_{1}^{,8}y\,dx+\int_{0}^{,6}x\,dy+\int_{1}^{0}2\,dz\right]$$

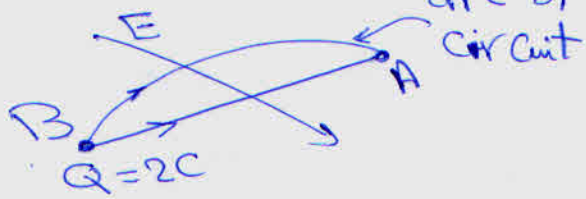
بالتعويض بمعادلة الدائرة عندما x ، y بما يعادلهم

$$=-2\left[\int_{1}^{,8}\sqrt{1-x^2}\,dx+\int_{0}^{,6}\sqrt{1-y^2}\,dy\right]+0$$

$$=-2\left[\frac{1}{2}\left(x\sqrt{1-x^2}+\sin^{-1}x\right)\Big|_{x=1}^{,8}+\left(y\sqrt{1-y^2}+\sin^{-1}y\right)\Big|_{0}^{,6}\right]$$

$$=-,96\ J$$

طب لو مشيت على خط مستقيم ← بدل ما هعوض عندما x ، y بمعادلة الدائرة هشوف ما يعادلهم في معادلة المستقيم



معادلة المستقيم :- $\dfrac{x-x_1}{y-y_1}=\dfrac{\Delta x}{\Delta y}=\dfrac{x_2-x_1}{y_2-y_1}$

$$\boxed{y=-3(x-1)}$$

$$w=-2\left[\int_{x=1}^{,8}-3(x-1)\,dx+\int_{0}^{,8}\left(\frac{-1}{3}y+1\right)dy+0\right]$$

$$=-6\left[\left(\frac{x^2}{2}-x\right)\Big|_{1}^{,8}\right]-2\left[\left(\frac{-1}{6}y^2+y\right)\Big|_{0}^{,6}\right]=-,96\ J$$

* الشغل المبذول لنقل شحنة Q في مجال كهربي $\bar{E}$ لا يعتمد على المسار الذي تنتقل من خلاله الشحنة.

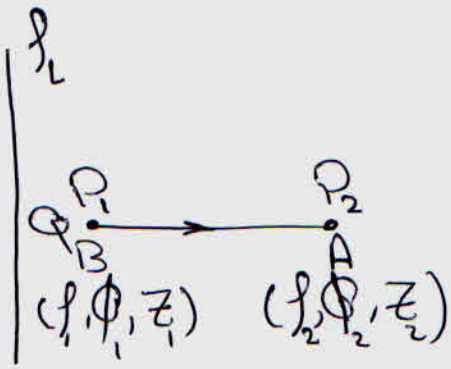
ex/

$$W_D = Q\int_{in}^{Fi} \bar{E}\cdot d\bar{L}$$

$$= -Q\int_{B}^{A} \bar{E}\cdot d\bar{l}$$

(cyl)

$$\therefore \bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}\bar{a_\rho}$$



$$\therefore d\bar{L} = d\rho\,\bar{a_\rho} + \rho\,d\phi\,\bar{a_\phi} + dz\,\bar{a_z}$$

$$\therefore W_D = -Q\int \left(\frac{\rho_L}{2\pi\epsilon\rho}\bar{a_\rho}\right)\cdot\left(d\rho\,\bar{a_\rho} + \rho\,d\phi\,\bar{a_\phi} + dz\,\bar{a_z}\right)$$

$$= \frac{-Q\rho_L}{2\pi\epsilon}\int_{\rho_1}^{\rho_2}\frac{1}{\rho}d\rho \quad (J)$$

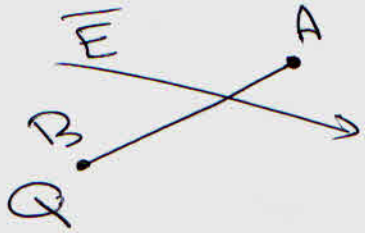
$$= \frac{-Q}{2\pi\epsilon}\ln\rho\Big|_{\rho_1}^{\rho_2} = \frac{-Q\rho_L}{2\pi\epsilon}\ln\frac{\rho_2}{\rho_1} \quad (J)$$

* Potential and Potential difference

هو الشغل المبذول لنقل وحدة الشحنات من النقطة الأولى إلى الثانية

$$W_D = -Q\int_{B}^{A}\bar{E}\cdot d\bar{L}$$

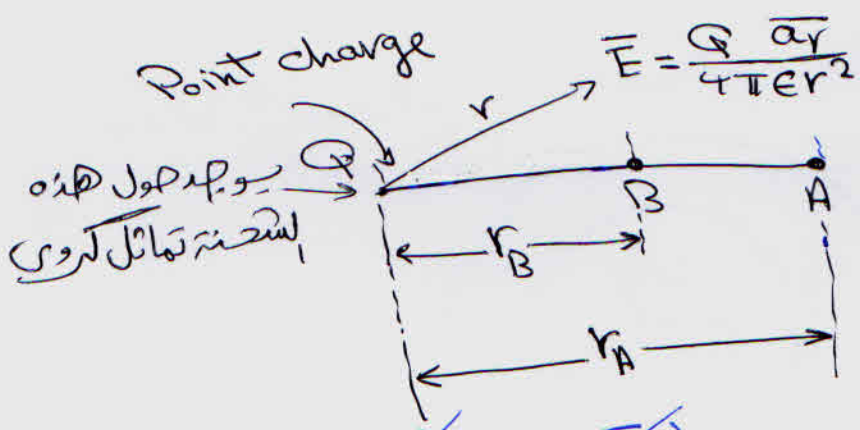
$$V_{AB} = V_A - V_B = -\int_{B}^{A}\bar{E}\cdot d\bar{L} \quad (V) \text{ or } \left(\frac{J}{C}\right)$$



For infinite line

$$W_D = \frac{-Q\rho_L}{2\pi\epsilon}\ln\left(\frac{\rho_2}{\rho_1}\right)$$

$$V_{AB} = \frac{W_D}{Q} = \frac{-\rho_L}{2\pi\epsilon}\ln\left(\frac{\rho_2}{\rho_1}\right) \quad (V)$$



$$V_{AB} = -\int_B^A \bar{E}\cdot d\bar{l}$$

$$= -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon r^2}\,\bar{a}_r\cdot(dr\,\bar{a}_r + \cdots \bar{a}\phi + \cdots a\bar{\theta})$$

$\bar{a}\phi \to 0$, $a\bar{\theta} \to 0$

$$= \frac{-Q}{4\pi\epsilon}\int_{r_B}^{r_A}\frac{dr}{r^2} = \frac{-Q}{4\pi\epsilon}\left[\frac{-1}{r}\right]_{r_B}^{r_A}$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

* ——— * ——— *

ال Potential عند نقطة هو الجهد المبذول لنقل وحدة الشحنات من اللانهاية إلى النقطة .

$$V_A = V_{AB}\Big|_{r_B=\infty}$$

$$= \frac{Q}{4\pi\epsilon r_A}\ (V)\quad (\text{Scaler})$$

$$V_\infty = 0$$

$$V_A = V_A - V_B = V_{AB}\Big|_{B=0\ at\ \infty}$$

مثال :-

$V_{AB} = V_A - V_B$

$$= -\int_B^A \bar{E} \cdot d\bar{L}$$

For Point charge :- $V_{AB} = \frac{Q}{4\pi\epsilon}\left(\frac{1}{R_A} - \frac{1}{R_B}\right)$

$$= -\oint_B^B \bar{E} \cdot d\bar{L} = 0 = V_B - V_B$$

$$\therefore \oint \bar{E} \cdot d\bar{L} = 0$$

Potential Gradiant :-

$$V = -\int E \cdot dL$$

$$\Delta V = -E \cdot \Delta L$$

* ΔV هو معدل تغير الـ Potential بالنسبة لـ electric field في الـ space

For $\theta = 0$

$$\left.\frac{-\Delta V}{\Delta L}\right|_{\text{For } \theta=0} = E$$

Cartizian Gordinate :-

$$\bar{E} = \left(\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right)$$

$$\bar{E} = -\nabla V$$

ex) $V = 2x^2y - 5z$ at $P = (-4, 3, 6)$

Find V, D, E at P.

V at P

$V = 2(-4)^2(3) - 5(6) =$ V

E at P

$\bar{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right)$

$= -\left[4xy\,\bar{a}_x + 2x^2\bar{a}_y - 5\,\bar{a}_z\right)$ V/m

نعوض بالنقطة

=

D at P

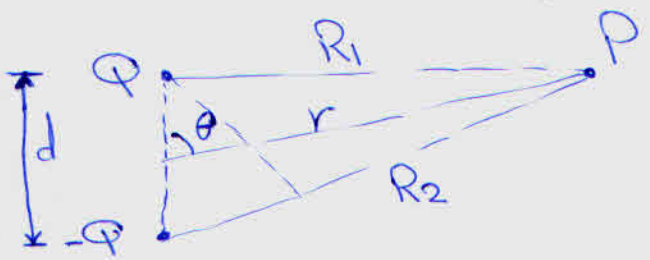
$\bar{D} = \epsilon_0 \bar{E}$

Electric dipole

شحنتين متساويتين بينهما مسافة صغيرة ومختلفتين في الاشارة

$V_P = \frac{Q}{4\pi\epsilon R_1} - \frac{Q}{4\pi\epsilon R_2}$

$= \frac{Q}{4\pi\epsilon}\frac{R_2 - R_1}{R_1 R_2}$



if P is very far

كلما كانت d أصغر كلما كان الخطأ أقل

$R_1 \simeq R_2 \simeq r$

$R_2 - R_1 = d\cos\theta$

* إذا أعطيت الفرق يعود الى خطأ كبير

$V_P = \frac{Q}{4\pi\epsilon}\frac{d\cos\theta}{r^2}$

$= \frac{P\cos\theta}{4\pi\epsilon r^2}$

where $\Rightarrow \boxed{P = Qd}$ → Electric moment

$$E = -\nabla V = -\left(\frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi\right)$$

$$= -\left[-\frac{2P\cos\theta}{4\pi\epsilon r^3}\bar{a}_r - \frac{P\sin\theta}{4\pi\epsilon r^3}\bar{a}_\theta\right]$$

$$= \frac{1}{4\pi\epsilon r^3}\left[2P\cos\theta\,\bar{a}_r + P\sin\theta\,\bar{a}_\theta\right]$$

Chapter 5 : Conductors and dielectrics.

* Currents :

1. Convection Current. $\Longrightarrow J_v = \rho_v V$
2. Conduction " . $\Longrightarrow J_c = \sigma \bar{E}$
3. Displacement " . $\Longrightarrow J_d = \epsilon \frac{\partial E}{\partial t}$

* Continuity equation :

$$I = \oint_S J \cdot ds = \frac{-dQ}{dt} = \frac{-d}{dt}\int \rho_v \, dv$$

Divergence theorem $\Longleftarrow$ picture

$$\int_v (\nabla \cdot J)\, dv = \frac{-d}{dt}\int_v \rho_v \, dv$$

$$\nabla \cdot \bar{J} = \frac{-\partial}{\partial t}\rho_v \quad \Longleftarrow \text{Continuity equation}$$

$$\left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right) = \frac{\partial \rho_v}{\partial t}$$